A design method for oversampled warped cosine-modulated filter banks

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***Abstract*— This paper presents an efficient method for the design of M-channel oversampled warped cosine-modulated filter banks. The method consists in filter prototype design for synthesis filter bank using optimization procedure, which minimizes filter bank overall distortion transfer function. Optimization takes into account channel subsampling factors as well.**

I. Introduction

Multirate digital filter banks are used extensively in many multimedia applications such as subband coders for speech signals and image processing [1], [2]. In these applications signal of interest is often decomposed into frequency subbands which can be processed in parallel at a lower sampling rate.

Modulated uniform filter banks (where all filters are shift versions of one filter prototype) have been well studied [3]. Nevertheless, in many applications a non-uniform time-frequency representation is more appropriate. For example, in hearing aids a filter bank approximating critical bands of human auditory system is highly desired. As was pointed out in [4] to reduce computational complexity of filter bank for hearing aids all subband signals should be subsampled. Another crucial requirement for filter bank for hearing aids is high attenuation of aliasing components. It follows from the fact that for hearing loss compensation a wide range of different gains applied to the different frequency bands.

In this paper oversampled warped cosine-modulated filter banks with low distortion level is considered. An efficient method for the design of filter prototype for synthesis filter is proposed. Method allow to obtain filter bank with nonuniform frequency partitioning, and high aliasing and imaging attenuation.

II. Warped cosine-modulated filter bank

The warped version of cosine-modulated filter banks (CMFB) has been introduced in [5] as alternative to nonuniform oversampled DFT filter banks [6]. The advantage of warped CMFB over [6] is that for real-valued input the subband signals are also real.

The transfer functions of the analysis and synthesis filters ($H_k(z)$ and $F_k(z)$ respectively) of an M-channel uniform cosine-modulated filter bank can be expressed as follows [7]:

$$\begin{aligned} H_k(z) &= a_k b_k H(zW_{2M}^{k+0.5}) + a_k^* b_k^* H(zW_{2M}^{-(k+0.5)}), \\ F_k(z) &= a_k^* b_k F(zW_{2M}^{k+0.5}) + a_k b_k^* F(zW_{2M}^{-(k+0.5)}), \end{aligned} \quad (1)$$

where $k = 0, 1, \ldots, M-1$ number of channel,

$$a_k = \quad e^{j(-1)^k \frac{\pi}{4}}, \quad b_k = \quad W_{2M}^{(k+0.5)\frac{N-1}{2}}, \quad (2)$$

and $W_{2M} = e^{-j\pi/M}$. The $H(z)$ and $F(z)$ are linear phase lowpass FIR filter prototypes of order N with cutoff frequency $\omega_c \approx \pi/2M$.

According to [5] wapred cosine-modulated filter bank is obtained from uniform one by replacement of all delay element with allpass filters:

$$z^{-1} \to A(z). \quad (3)$$

Substitution (3) is referred to as *allpass transform* [8]–[9]. In this paper we consider stable and causal first-order allpass filter, which is defined as

$$A(z) = \frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}, \quad \alpha \in \mathbb{R}, \quad |\alpha| < 1, \quad (4)$$

with the frequency response $A(e^{j\omega}) = e^{j\varphi(\omega)}$. The phase response is written as

$$\varphi(\omega) = -\omega + 2\arctan\left(\frac{\alpha \sin\omega}{\alpha\cos\omega - 1}\right). \quad (5)$$

Replacing all terms z^{-1} by allpass elements results in transformation of the frequency scale $\omega \to \varphi(\omega)$. This transformation is illustrated in fig. 1.

Let us consider of nonuniform oversampling analysis-synthesis filter bank as depicted in fig. 2. In our case $H_k(z)$

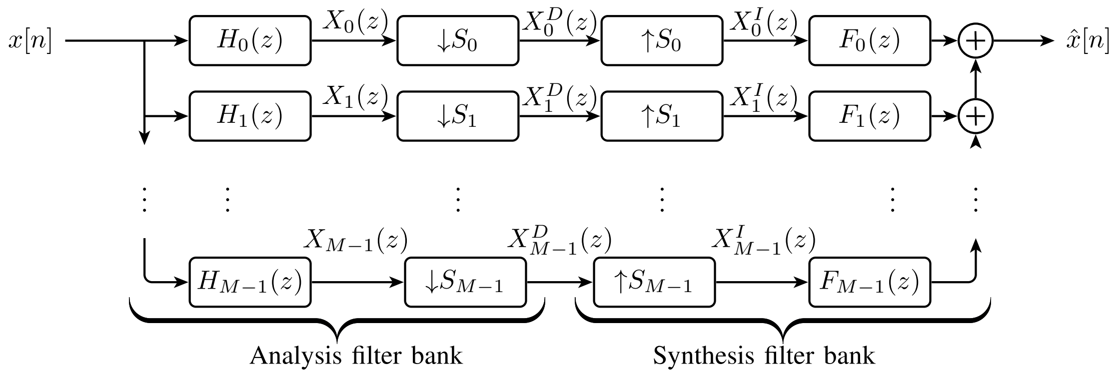


Fig. 2. Nonuniform analysis-synthesis filter bank

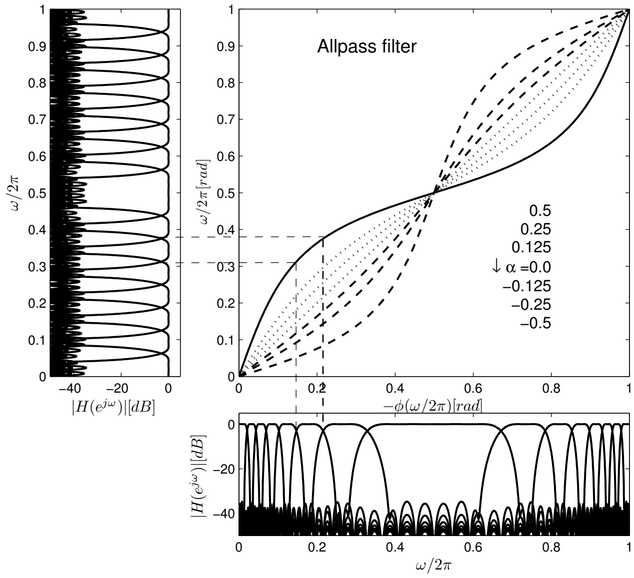


Fig. 1. Allpass transformation of warped CMFB

and $F_k(z)$ are allpass transformed version of (1) and S_k the subsampling factor for the k-th subband. As we consider nonuniform filter bank the subsampling fastors S_k can be different.

The input signal in fig. 2 $x[n]$ is passed through the analysis filter bank with consequent downsampling by a factor of S_k yielding the frequency response of subband signal:

$$X_k^D(e^{j\omega}) = \frac{1}{S_k}\sum_{i=0}^{S_k-1}\left[d_k H\left(e^{-j\varphi(\frac{\omega+2\pi i}{S_k})}W_{2M}^{(k+0.5)}\right)\right. \quad (6)$$
$$\left. +\hat{d}_k H\left(e^{-j\varphi(\frac{\omega+2\pi i}{S_k})}W_{2M}^{-(k+0.5)}\right)\right] \times X(e^{-j\varphi(\frac{\omega+2\pi i}{S_k})}),$$

where $d_k = a_k b_k$, $\hat{d}_k = a_k^* b_k^*$. After subband signals interpolation by factor S_k the frequency response can be written as

$$X_k^I(e^{j\omega}) = \sum_{i=0}^{S_k-1}\left[d_k H\left(e^{-j\varphi(\omega+\frac{2\pi i}{S_k})}W_{2M}^{(k+0.5)}\right)\right. \quad (7)$$
$$\left. +\hat{d}_k H\left(e^{-j\varphi(\omega+\frac{2\pi i}{S_k})}W_{2M}^{-(k+0.5)}\right)\right] \times X(e^{-j\varphi(\omega+\frac{2\pi i}{S_k})}).$$

By summation of filtered $X_k^I(e^{j\omega})$ signals we obtain reconstructed signal

$$\hat{X}(e^{j\omega}) = \sum_{k=0}^{M-1}\sum_{i=0}^{S_k-1}\left[d_k H\left(e^{-j\varphi(\omega+\frac{2\pi i}{S_k})}W_{2M}^{(k+0.5)}\right)\right. \quad (8)$$
$$\left. +\hat{d}_k H\left(e^{-j\varphi(\omega+\frac{2\pi i}{S_k})}W_{2M}^{-(k+0.5)}\right)\right] \times$$
$$\left[c_k F\left(e^{-j\varphi(\omega)}W_{2M}^{(k+0.5)}\right) + \hat{c}_k F\left(e^{-j\varphi(\omega)}W_{2M}^{-(k+0.5)}\right)\right] \times$$
$$X(e^{-j\varphi(\omega+2\pi i/S_k)}),$$

where $c_k = a_k^* b_k$, $\hat{c}_k = a_k b_k^*$. Introduce the notation

$$\begin{aligned}
\gamma_1(\omega,i,k) &= -\varphi(\omega+2\pi i/S_k) + 2\pi(k+0.5)/2M, \quad (9)\\
\gamma_2(\omega,i,k) &= -\varphi(\omega+2\pi i/S_k) - 2\pi(k+0.5)/2M,\\
\psi_1(\omega,k) &= -\varphi(\omega) + 2\pi(k+0.5)/2M,\\
\psi_2(\omega,k) &= -\varphi(\omega) - 2\pi(k+0.5)/2M,\\
H_{k,i}(\omega) &= d_k H\left(e^{-j\gamma_1(\omega,i,k)}\right) + \hat{d}_k H\left(e^{-j\gamma_2(\omega,i,k)}\right),\\
F_k(\omega) &= c_k F\left(e^{-j\psi_1(\omega,k)}\right) + \hat{c}_k F\left(e^{-j\psi_2(\omega,k)}\right).
\end{aligned}$$

Using (9) we can express overall distortion filter bank frequency response $T_{\text{all}}(e^{j\omega})$, which is sum of aplitude and aliasing distortions

$$T_{\text{all}}(e^{j\omega}) = T_{\text{dist}}(e^{j\omega}) + T_{\text{alias}}(e^{j\omega}), \quad (10)$$

where

$$T_{\text{dist}}(e^{j\omega}) = \sum_{k=0}^{M-1} H_{k,0}(\omega)F_k(\omega),$$
$$T_{\text{alias}}(e^{j\omega}) = \sum_{k=0}^{M-1}\sum_{i=1}^{S_k-1} H_{k,i}(\omega)F_k(\omega).$$

III. Synthesis filter bank design

A. Problem formulation

Our goal is to design filter prototype for synthesis filter bank that suppresses aliasing/imaging components caused by decimation/interpolation. We assume that filter prototype for analysis filter bank is fixed and designed using techniques for uniform filter banks (for example [10]).

The optimization problem for the design of the warped cosine-modulated synthesis filter bank is formulated as follows:

$$\min |E(\omega)|, \quad \forall\omega \in [0,\, \pi]. \tag{11}$$

The error function $E(\omega)$ is defined as

$$E(\omega) = \left|T_{\text{all}}(e^{j\omega})\right|^2 - 1, \quad \omega \in [0,\, \pi]. \tag{12}$$

A weighted least square objective function (13) is formulated to solve problen in (11)

$$g = \sum_{\omega\in[0,\, \pi]} B(\omega)|E(\omega)|^2, \tag{13}$$

where $B(\omega)$ is a cost weighting function.

B. The design method

Proposed method is used weighting least square optimization (similar to that is used in [11]) and consist of two iterative procedures: one nested within the other. The prototype filter coefficients $f_l[n]$ are iteratively optimized in inner loop procedure, while in the outer loop procedures the cost weighting $B_\mu(\omega)$ function is updated. During inner loop procedure $B_\mu(\omega)$ kept constant. Here l and μ are running indexes for inner and outer loop procedures respectively.

At the l-th iteration of inner loop frequency response of synthesis filter-prototype $F_l(\omega)$ is written as [11]

$$F_l(\omega) = e^{j[(N-1)\omega/2]}\mathbf{C}^T(\omega)\mathbf{f}_l, \tag{14}$$

where

$$\mathbf{C}(\omega) = \begin{cases} \left[2\cos\left(\frac{\omega}{2}\right)\ 2\cos\left(\frac{3\omega}{2}\right)\ldots 2\cos\left(\frac{[N-1]\omega}{2}\right)\right]^T, \\ N \text{ even}, \\ \left[1\ 2\cos(\omega)\ldots 2\cos\left(\frac{[N-1]\omega}{2}\right)\right]^T, \\ N \text{ odd}, \end{cases} \tag{15}$$

$$\mathbf{f}_l = \left[f_l\left(\left\lfloor\frac{N}{2}\right\rfloor\right)\ f_l\left(\left\lfloor\frac{N}{2}\right\rfloor+1\right)\ldots f_l\,(N-1)\right]. \tag{16}$$

The operator $\lfloor a \rfloor$ denotes the largest integer smaller than a.

Using (9) and (14) the error function (12) at the l-th iteration can be expressed as follows

$$E_l(\omega) = \mathbf{f}_l^T\big(\mathbf{P}_1(\omega) + \mathbf{P}_2(\omega)\big)\mathbf{f}_l - 1, \tag{17}$$

where

$$\mathbf{P}_1(\omega) = \mathbf{Q}_1^T(\omega)\mathbf{Q}_1(\omega), \quad \mathbf{P}_2(\omega) = \mathbf{Q}_2^T(\omega)\mathbf{Q}_2(\omega), \tag{18}$$

$$\mathbf{Q}_1(\omega) = \sum_{k=0}^{M-1} |V_{S_k}(\omega)|\Big[\cos\big(\phi_{1,k}(\omega) + \arg(V_{S_k}(\omega))\big) \times \mathbf{C}^T(\psi_1(\omega)) + \cos\big(\phi_{2,k}(\omega) + \arg(V_{S_k}(\omega))\big)\mathbf{C}^T(\psi_2(\omega))\Big], \tag{19}$$

$$\mathbf{Q}_2(\omega) = \sum_{k=0}^{M-1} |V_{S_k}(\omega)|\Big[\sin\big(\phi_{1,k}(\omega) + \arg(V_{S_k}(\omega))\big) \times \mathbf{C}^T(\psi_1(\omega)) + \sin\big(\phi_{2,k}(\omega) + \arg(V_{S_k}(\omega))\big)\mathbf{C}^T(\psi_2(\omega))\Big], \tag{20}$$

$$V_{S_k}(\omega) = \sum_{i=0}^{S_k-1} H_{k,i}(\omega), \tag{21}$$

and

$$\phi_{1,k}(\omega) = -(-1)^k\tfrac{\pi}{4} - 2\pi\tfrac{N-1}{2}(k+0.5)/2M - \psi_1(\omega)/2,$$
$$\phi_{2,k}(\omega) = \ \ (-1)^k\tfrac{\pi}{4} - 2\pi\tfrac{N-1}{2}(k+0.5)/2M - \psi_2(\omega)/2.$$

Thereby the error function (12) is presented as quadratic function (17), which simplifies following optimization.

The necessary condition for minimization of (13) is equality of gradient $\bigtriangledown g$ to zero. Thus the gradient of objective function (13) at l-th iteration is obtained by substituting (17) into (13) and differentiating it with respect to $\mathbf{f}$

$$\bigtriangledown g_l = 2\sum_{\omega\in[0,\, \pi]} B_\mu(\omega)\left(\mathbf{f}_l^T\mathbf{P}_{1,2}(\omega)\mathbf{f}_l - 1\right) \times \left[\mathbf{P}_{1,2}(\omega) + \mathbf{P}_{1,2}^T(\omega)\right]\mathbf{f}_l, \tag{22}$$

where $\mathbf{P}_{1,2}(\omega) = \mathbf{P}_1(\omega) + \mathbf{P}_2(\omega)$. Let $\mathbf{f}_{l,opt} = \mathbf{f}_l + \mathbf{e}_l$ denote optimum solution that sets (22) to zero. Then expanding $\bigtriangledown g_l$ in Taylor's series

$$\bigtriangledown g_l + \bigtriangledown^2 g_l\mathbf{e}_l + \ldots = 0, \tag{23}$$

where $\bigtriangledown^2 g_l$ matrix of second partial derivatives of (13) with respect to $\mathbf{f}$ is given by

$$\bigtriangledown^2 g_l = 2\sum_{\omega\in[0,\, \pi]} B_\mu(\omega)\big((\mathbf{f}_l^T\mathbf{P}_{1,2}(\omega)\mathbf{f}_l - 1)\mathbf{P}_{1,2}(\omega) + \big(\left[\mathbf{P}_{1,2}(\omega) + \mathbf{P}_{1,2}^T(\omega)\right]\mathbf{f}_l\big) \times \big(\left[\mathbf{P}_{1,2}(\omega) + \mathbf{P}_{1,2}^T(\omega)\right]\mathbf{f}_l\big)^T\big). \tag{24}$$

Discarding the higher order terms in (23), the correction vector $\mathbf{e}_l$ obtained by solving

$$\mathbf{e}_l = -(\bigtriangledown^2 g_l)^{-1} \bigtriangledown g_l. \tag{25}$$

Initialy coefficients $f[n]$ are taken equal to $h[n]$. At the lth iteration, the vector $\mathbf{e}_l$ is obtained using (25). The coefficient vector is updated as follows

$$\mathbf{f}_{l+1} = \mathbf{f}_l + \mathbf{e}_l \tag{26}$$

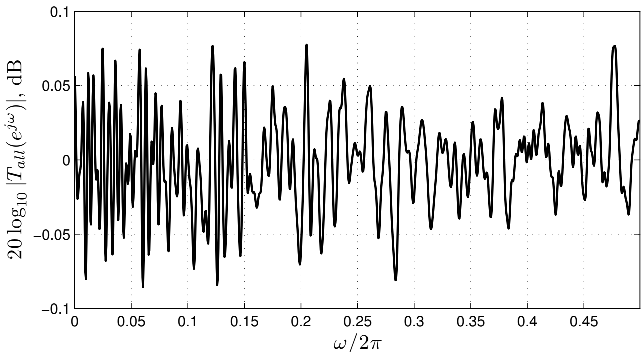
and inner loop procedure terminates when $||\mathbf{e}_l||^2 \le 10^{-10}$.

The outer procedure updates cost weighting function $B_\mu(\omega)$ using the algorithm described in [11].

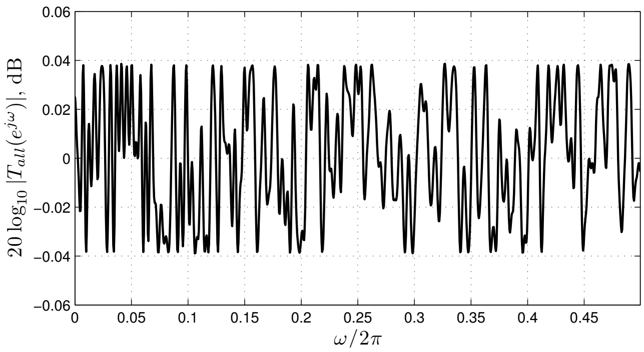
IV. Design examples

We provide design example to illustrate the proposed algorithm. To measure the performance of our design we used overall distortion filter bank magnitude response $20\log_{10}\left|T_{\text{all}}(e^{j\omega})\right|$.

Let us consider the 18-channel warped CMFB approximating the psychoacoustic Bark scale for standard sampling frequency of 8 kHz (according to [12] warping coefficient $\alpha = 0.4092$) with subsampling ratios $S_k = \{12, 18, 12, 9, 7, 11, 8, 8, 10, 6, 5, 4, 8, 3, 1, 2, 3, 4\}$. Practical rule of selection subsampling ratios is described in [13]. Filter prototype $H(e^{j\omega})$ of order $N = 144$ for filter bank under consideration is designed using method [11]. In fig. 3-a overall distortion transfer function magnitude response of warped CMFB is depicted for the case when $f[n] = h[n]$, while in fig. 3-b the same function given for the case when coefficients $f[n]$ optimized using algorithm described in sec.III. In fig. 4 analysis and synthesis prototype filters magnitude responses of optimized warped CMFB are given.



(a) Distortion function of initial warped CMFB



(b) Distortion function of optimized warped CMFB

Fig. 3. Magnitude responses of distortion functions $20\log_{10}\left|T_{\text{all}}(e^{j\omega})\right|$

It is evident from the fig. 3 that proposed method allows significantly reduce distortion in warped oversampled CMFB. Peak distortion value decreases from 0.0775 dB to 0.0386 dB.

V. Conclusions

In this paper a method for design of synthesis filter prototype has been proposed. It can be used to obtain oversampled

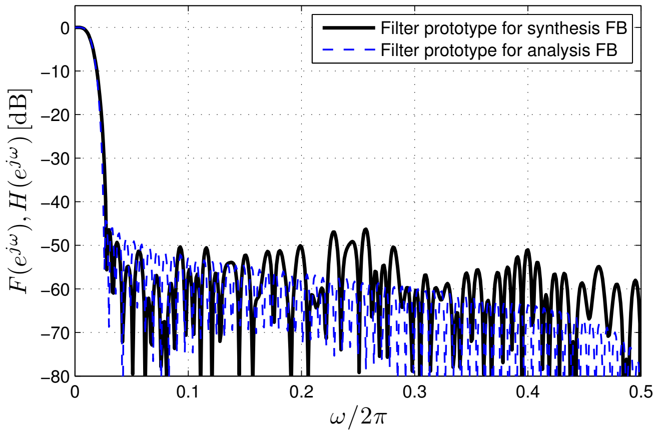


Fig. 4. Magnitude responses of filter prototypes for synthesis and analysis filter banks

warped cosine-modulated filter bank with high aliasing and imaging attenuation.

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